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The influences of recycle on a double-pass laminar counterflow concentric circular heat exchangers

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Abstract

A new device of inserting an impermeable sheet with negligible thermal resistance to divide a circular tube into two subchannels with uniform wall temperature and external refluxes at the ends, resulting in substantially improving the heat transfer, has been designed. The mathematical formulation and theoretical analysis to such a conjugated Graetz problem of double-pass concentric circular heat exchangers have been developed by the use of an orthogonal expansion technique. The analytical results are represented graphically and compared with that in an open conduit of the same size without recycle. Considerable improvement in heat transfer is obtainable by employing double-pass operations with inserting an impermeable sheet instead of using single-pass operations without external refluxes. Two numerical examples in heat transfer efficiency by arranging the recycle effect as well as the power consumption were illustrated. The effects of the channel thickness ratio on the enhancement of heat transfer efficiency as well as on the power consumption increment have been also discussed.

Keywords: Heat exchangers; Double-pass operations; Concentric circular tubes; Recycle

1. Introduction

The Graetz problem [1,2] is considered to be that of heat transfer to fully developed laminar flow in a bounded conduit. There are a number of other heat- and mass-transfer processes that are closely related to the Graetz problem. The classical Graetz problem has been extended to deal with multistream heat- and mass-transfer problems and conjugated boundary value problems of several types [3–9]. The applicability of double-pipe heat exchangers [10] is used broadly in industrial applications and the recycling of the fluid at the ends has been designed and developed in many separation processes and reactor design, such as loop reactors [11,12], air-lift reactors [13,14] and draft-tube bubble columns [15,16]. The double-flow and recycle-effect concepts can effectively enhance the heat transfer rate and lead to improved performance in designing devices. However, a solution for such a counterflow problem in concentric heat exchangers with external refluxes does not follow readily from either the single-pass device or the cocurrent device because of the interfacial boundary condition in the adjacent phases.

The purposes of the present study are to extend the previous works [17,18] to the heat transfer in concentric heat exchangers with external refluxes, and develop the mathematical formulations and obtain the analytical solution by the use of an orthogonal expansion technique [19–26] with the eigenfunction expanding in terms of an extended power series. This work includes the influence of recycling on heat transfer and the transfer efficiency improvement with reflux ratio and Graetz number as parameters.

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Nomenclature

C_p	heat capacity, J/kg k
D	hydraulic radius, m
F_m	eigenfunction associated with eigenvalue λ_m
f	friction factor
g_c	conversion factor, kg m/s^2 N
Gz	Graetz number, $4V/\alpha\pi L$
G_m	function defined during the use of orthogonal expansion method
$\bar{h}$	average heat transfer coefficient, kW/m^2 K
h_{fs}	friction loss in conduit, m^2/s^2
I_h	heat transfer improvement, defined by Eq. (25)
I_p	power consumption increment, defined by Eq. (29)
k	thermal conductivity of the fluid, kW/m K
L	conduit length, m
M	reflux ratio, reverse volume flow rate divided by input volume flow rate
$\overline{Nu}$	Nusselt number
P_0	hydraulic dissipated energy
$2R$	inside diameter of the outer tube, m
Re	Reynolds number
S_m	expansion coefficient associated with eigenvalue λ_m
T	temperature of fluid, K
V	input volume flow rate of conduit, m^3/s
v	velocity distribution of fluid, m/s
$\bar{v}$	average velocity of fluid, m/s
r	radial coordinate, m
z	axial coordinate, m

Greek symbols

α	thermal diffusivity of fluid, m^2/s
δ	thickness of the impermeable sheet, m
ξ	longitudinal coordinate, z/L
η	transversal coordinate, r/R
θ	dimensionless temperature $(T-T_i)/(T_w-T_i)$
κ	channel thickness ratio
λ_m	eigenvalue
μ	fluid viscosity, kg/m s
ρ	fluid density, kg/m^3
ψ	dimensionless temperature $(T-T_w)/(T_i-T_w)$

Subscripts

a	in forward flow channel
b	in backward flow channel
fs	friction loss on the surface
F	at the outlet of a double-pass device
i	at the inlet
L	at the outlet, $\xi = 1$
0	in a single-pass device without recycle
w	at the wall surface

2. Theoretical formulations

2.1. Temperature distributions in concentric tubes of double-pass devices with recycle

Consider a circular tube of length L and inside diameter $2R$ which is divided into two subchannels with thickness $2\kappa R$ and $2(1-\kappa)R$, respectively, by inserting an impermeable sheet of negligible thickness and thermal resistance, as shown in Fig. 1. Before entering the annulus for a double-pass operation as shown in Fig. 1, the fluid with volume flow rate V and the inlet temperature T_i from the inner tube, will mix with the fluid of volume flow rate MV and the outlet temperature T_F, which is regulated by using a conventional pump. The fluid is completely mixed at the inlet and outlet of the tube.

After the following assumptions are made in the present analysis: constant physical properties and wall temperatures of the outer tubes; purely fully developed laminar flow on the entire length in each subchannel; negligible end effect, axial conduction and thermal resistance of the impermeable sheet; the velocity distributions and equations of energy in dimensionless form may be obtained as

$$\frac{v_a(\eta)R^2}{\alpha L}\frac{\partial\psi_a(\eta,\xi)}{\partial\xi} = \frac{1}{\eta}\left[\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\psi_a(\eta,\xi)}{\partial\eta}\right)\right] \tag{1}$$

$$\frac{v_b(\eta)R^2}{\alpha L}\frac{\partial\psi_b(\eta,\xi)}{\partial\xi} = \frac{1}{\eta}\left[\frac{\partial}{\partial\eta}\left(\eta\frac{\partial\psi_b(\eta,\xi)}{\partial\eta}\right)\right] \tag{2}$$

$$v_a(\eta) = 2\bar{v}_a\left(1-\left(\frac{\eta}{\kappa}\right)^2\right), \quad 0\leqslant\eta\leqslant\kappa \tag{3}$$

$$v_b(\eta) = \frac{2\bar{v}_b}{\left[\frac{1-\kappa^4}{1-\kappa^2}-\frac{1-\kappa^2}{\ln 1/\kappa}\right]}\left[1-\eta^2+\left(\frac{1-\kappa^2}{\ln 1/\kappa}\right)\ln\eta\right], \quad \kappa\leqslant\eta\leqslant 1 \tag{4}$$

$$\bar{v}_a = \frac{V}{\pi(\kappa R)^2}, \quad \bar{v}_b = -\frac{(M+1)V}{\pi R^2-\pi(\kappa R)^2} \tag{5}$$

in which

$$\eta = \frac{r}{R}, \quad \xi = \frac{z}{L}, \quad \psi_a = \frac{T_a-T_w}{T_i-T_w}, \quad \psi_b = \frac{T_b-T_w}{T_i-T_w}, \quad \theta_a = 1-\psi_a = \frac{T_a-T_i}{T_w-T_i}, \quad \theta_b = 1-\psi_b = \frac{T_b-T_i}{T_w-T_i}, \quad Gz = \frac{4V}{\alpha\pi L} \tag{6}$$

The boundary conditions for solving Eqs. (1) and (2) are

$$\frac{\partial \psi_a(0,\xi)}{\partial \eta} = 0 \tag{7}$$

$$\psi_b(1,\xi) = 0 \tag{8}$$

$$\frac{\partial \psi_a(\kappa,\xi)}{\partial \eta} = \frac{\partial \psi_b(\kappa,\xi)}{\partial \eta} \tag{9}$$

$$\psi_a(\kappa,\xi) = \psi_b(\kappa,\xi) \tag{10}$$

and the dimensionless outlet temperature is

$$\theta_F = 1 - \psi_F = \frac{T_F - T_i}{T_w - T_i} \tag{11}$$

2.2. Temperature distributions in the single-pass device

For the single-flow device of the same size without recycle the impermeable sheet in Fig. 1 is removed, as shown in Fig. 2 and thus, $\kappa = 1$. The velocity distribution and equation of energy in dimensionless form may be written as

$$\frac{v_0(\eta)R^2}{\alpha L}\frac{\partial \psi_0(\eta,\xi)}{\partial \xi} = \frac{1}{\eta}\left[\frac{\partial}{\partial \eta}\left(\eta \frac{\partial \psi_0(\eta,\xi)}{\partial \eta}\right)\right] \tag{12}$$

$$v_0(\eta) = 2\bar{v}_0(1-\eta^2)\ 0 \leqslant \eta \leqslant 1 \tag{13}$$

$$\eta = \frac{r}{R},\quad \xi = \frac{z}{L},\quad \psi_0 = \frac{T_0 - T_w}{T_i - T_w},$$
$$\theta_0 = 1 - \psi_a = \frac{T_0 - T_i}{T_w - T_i},\quad Gz = \frac{4V}{\alpha \pi L},\quad \bar{v}_0 = \frac{V}{\pi R^2} \tag{14}$$

The boundary conditions for solving Eq. (12) are

$$\frac{\partial \psi_0(0,\xi)}{\partial \eta} = 0 \tag{15}$$

$$\psi_0(1,\xi) = 0 \tag{16}$$

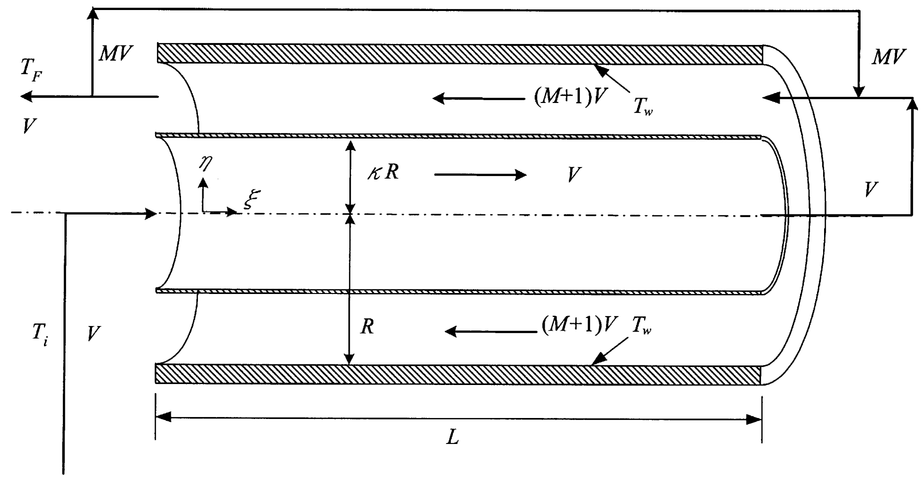

Fig. 1. Schematic diagrams of concentric circular heat exchangers with external refluxes at both ends.

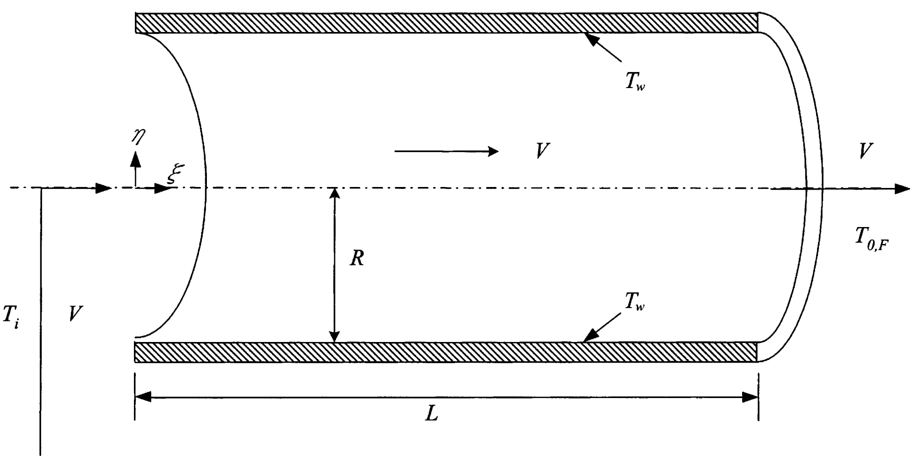

Fig. 2. Single-pass circular heat exchangers without recycle.

$$\psi_0(\eta_0, 0) = 1 \tag{17}$$

and the dimensionless outlet temperature is

$$\theta_{0,F} = 1 - \psi_F = \frac{T_{0,F} - T_i}{T_w - T_i} \tag{18}$$

3. Improvement of transfer efficiency

The Nusselt number for a double-pass operation with external refluxes may be defined as

$$\overline{Nu} = \frac{\bar{h}(2R)}{k} = \frac{\bar{h}D}{k} \tag{19}$$

in which the average heat transfer coefficient is defined as

$$q = \bar{h}2\pi RL(T_w - T_i) \tag{20}$$

Since

$$\bar{h}2\pi RL(T_w - T_i) = V\rho C_p(T_F - T_i) \tag{21}$$

or

$$\bar{h} = \frac{V\rho C_p(T_F - T_i)}{2\pi RL(T_w - T_i)} = \frac{V\rho C_p}{2\pi RL}(1 - \psi_F) \tag{22}$$

thus

$$\overline{Nu} = \frac{V}{\pi\alpha L}(1 - \psi_F) = \frac{1}{4}Gz(1 - \psi_F) \tag{23}$$

Similarly, for a single-pass operation without external refluxes

$$\overline{Nu}_0 = \frac{V}{\pi\alpha L}(1 - \psi_{0,F}) = \frac{1}{4}Gz(1 - \psi_{0,F}) \tag{24}$$

The improvement of performance by employing a double-pass operation with recycle is best illustrated by calculating the percentage increase in heat-transfer rate, based on the heat transfer of a single-pass operation with same device dimension and operating conditions, but without impermeable sheet and recycle, as

$$I_h = \frac{\overline{Nu} - \overline{Nu}_0}{\overline{Nu}_0} = \frac{\psi_{0,F} - \psi_F}{1 - \psi_{0,F}} = \frac{\theta_F - \theta_{0,F}}{\theta_{0,F}} \tag{25}$$

4. Increment of power consumption

The friction loss in conduits may be estimated by

$$h_{fs} = 4f\frac{L}{D}\frac{\bar{v}^2}{2g_c} \tag{26}$$

where $\bar{v}$ and D denote the bulk velocity in the conduit and the diameters of the conduit, respectively, while f is the friction factor which is the function of Reynolds number, Re. The friction loss in conduit of a single-pass device was calculated by $P_0 = V\rho h_{fs,0}$. Accordingly,

$$D_0 = 2R, D_a = 2\kappa R, \quad D_b = 2(R - \kappa R), \quad \bar{v}_0 = \frac{V}{\pi R^2} \tag{27}$$

$$\bar{v}_a = \frac{V}{\pi(\kappa R)^2}, \quad \bar{v}_b = \frac{V(M+1)}{\pi R^2 - \pi(\kappa R)^2} \tag{28}$$

and we have $f = 16/Re$ for the laminar flow in tube and the power consumption increment, may be defined as $I_p = (P - P_0)/P_0$, where $P = V\rho[h_{fs,a} + (M+1)h_{fs,b}]$. Therefore, the power consumption increment in double-pass devices is as follows:

$$I_p = \frac{1}{\kappa^4} + \frac{(M+1)^2}{(1-\kappa^2)(1-\kappa)^2} - 1 \tag{29}$$

Table 1
The power consumption increment with reflux ratio and channel thickness ratio as parameters

M	I_p		
	$\kappa = 0.3$	$\kappa = 0.5$	$\kappa = 0.7$
0.5	127	27	52
1.0	131	36	90
3.0	158	100	351
5.0	203	207	787

Some results for I_p of double-pass devices are presented in Table 1. It is seen from Table 1 that the power consumption increment does not depend on Graetz number but increases with the reflux ratio or as κ goes away from 0.5, especially for $\kappa > 0.5$.

5. Numerical examples

5.1. Power consumption in the single- and double-pass devices

As an illustration, the power consumption of the single-pass device will be illustrated by the working dimensions as follows: $L = 1.2$ m, $R = 0.2$ m, $V = 1 \times 10^{-4}$ m^3/s, $\mu = 8.94 \times 10^{-4}$ kg/m s, $\rho = 997.08$ kg/m^3. From those numerical values, the friction loss in conduit of a single-pass device was calculated by the appropriate equations and the result is

$$P_0 = V\rho h_{fs,0} = 1.71 \times 10^{-8} W = 2.29 \times 10^{-11} hp \tag{30}$$

5.2. Case studies on the heat transfer efficiency

The heat-transfer efficiency improvement by arranging the recycle effect will be illustrated by the following two case studies. Consider the heat transfer for a fluid flowing through a double-pass concentric circular tube with $R = 0.2$ m and $\kappa = 0.5$.

Table 2
The results of Case 1

$L \times 10^2$ (m)	$V \times 10^6$ (m^3/s)	Gz	Nu_0	I_h (%)		
				$M = 0.5$	$M = 3$	$M = 5$
16.7	1	50	3.17	−25.56	150.69	223.26
	5	250	3.55	−24.86	321.11	297.56
	10	500	3.60	−24.69	366.45	733.06
55.7	1	15	2.34	−24.86	42.56	56.87
	5	75	3.32	−25.33	197.54	309.47
	10	150	3.48	−24.97	274.69	476.96
83.5	1	10	1.92	−22.79	22.32	29.47
	5	50	3.17	−25.56	150.69	223.26
	10	100	3.40	−25.19	230.64	377.09

Table 3
The results of Case 2

$L \times 10^2$ (m)	$V \times 10^6$ (m^3/s)	Gz	Nu_0	I_h (%)		
				$M = 0.5$	$M = 3$	$M = 5$
14.44	1	100	3.40	−25.19	230.64	377.09
	5	500	3.60	−24.69	366.45	733.06
	10	1000	3.63	−24.69	392.96	823.36
57.77	1	25	2.77	−25.63	80.49	110.86
	5	125	3.45	−25.12	255.24	431.52
	10	250	3.55	−24.86	321.11	297.56
96.28	1	15	2.34	−24.86	42.56	56.87
	5	75	3.32	−25.33	197.54	309.47
	10	150	3.48	−24.97	274.69	476.96

Case 1: Pure water of 62 °C is flowing through the counterflow concentric circular tubes with constant wall temperature of 16 °C. The numerical values are assigned as

$$T_i = 62\ ^\circ\mathrm{C}, \quad T_w = 16\ ^\circ\mathrm{C},$$
$$\alpha_{water} = 1.524 \times 10^{-7}\ \mathrm{m^2/s}$$

Case 2: Oil at 62 °C is flowing through the counterflow concentric circular tubes with constant wall temperature of 16 °C. The numerical values are assigned as

$$T_i = 62\ ^\circ\mathrm{C}, \quad T_w = 16\ ^\circ\mathrm{C},$$
$$\alpha_{oil} = 8.816 \times 10^{-8}\ \mathrm{m^2/s}$$

From these values, the transfer efficiency improvements in laminar counterflow concentric circular tubes operated with external refluxes under various flow rates of fluid and reflux ratios, were calculated by the appropriate equations and the results are presented in Tables 2 and 3.

6. Results and discussion

By following the same mathematical treatment performed in parallel-plate heat exchangers of single- and double-pass devices [17], except the type of a concentric circular device and flow pattern, as shown in Fig. 1, with the eigenvalues $(\lambda_1, \lambda_2, \ldots, \lambda_m, \ldots)$ calculated from the following equations:

$$\frac{F'_{a,m}(\kappa)}{F_{a,m}(\kappa)} = \frac{F'_{b,m}(\kappa)}{F_{b,m}(\kappa)} \tag{31}$$

the results will be delineated as follows.

6.1. Dimensionless outlet temperatures in single- and double-pass devices

The outlet temperature for double-pass devices (θ_F) as well as for single-pass devices $(\theta_{0,F})$ were also obtained in terms of the Graetz number (Gz), eigenvalues (λ_m and $\lambda_{0,m}$), expansion coefficients ($S_{a,m}, S_{b,m}$ and $S_{0,m}$), location of impermeable sheet (κ) and eigenfunctions ($F_{a,m}(\eta_a), F_{b,m}(\eta_b)$ and $F_{0,m}(\eta_0)$). The results are

$$\begin{aligned}\psi_{0,F} &= \frac{2\pi\alpha L}{V}\sum\frac{S_{0,m}}{\lambda_m}\left(\int_0^1\left(F''_{0,m}\eta + F'_{0,m}\right)\mathrm{d}\eta\right)\\ &= \frac{8}{Gz}\sum\frac{S_{0,m}}{\lambda_m}1F'_{0,m}(1)\end{aligned} \tag{32}$$

or may be also obtained from the following overall energy balance on the outer tube for single-pass operation

$$\theta_{0,\mathrm{F}} = 1 - \psi_{0,\mathrm{F}} = \int_0^1 \frac{\alpha 2\pi L}{V}\left(-\frac{\partial \psi_{0,m}(1,\xi)}{\partial \eta}\right)\mathrm{d}\xi = \frac{8\lambda_{0,m}}{Gz}\sum_{m=0}^{\infty} S_{0,m}F'_{0,m}(1)(1-\mathrm{e}^{-\lambda_m}) \quad (33)$$

and the dimensionless outlet temperature ψ_F, which is referred to as the bulk temperature for double-pass operations, may be calculated by

$$\psi_\mathrm{F} = -\frac{\int_\kappa^1 v_\mathrm{b} 2\pi R^2 \eta \psi_\mathrm{b}(\eta,0)\,\mathrm{d}\eta}{V(M+1)} = -\frac{2\pi\alpha L}{V(M+1)}\sum \frac{\mathrm{e}^{-\lambda_m}S_{\mathrm{b},m}}{\lambda_m}\left(\int_\kappa^1 (F''_{\mathrm{b},m}+F'_{\mathrm{b},m})\,\mathrm{d}\eta\right) = -\frac{8}{Gz(M+1)}\sum \frac{\mathrm{e}^{-\lambda_m}S_{\mathrm{b},m}}{\lambda_m}[F'_{\mathrm{b},m}(1)-\kappa F'_{\mathrm{b},m}(\kappa)] \quad (34)$$

and may be examined by Eq. (35) which is readily obtained from the following overall energy balance on outer tube

$$\theta_\mathrm{F} = 1 - \psi_\mathrm{F} = \int_0^1 \frac{\alpha 2\pi L}{V}\left(-\frac{\partial \psi_{\mathrm{b},m}(1,\xi)}{\partial \eta}\right)\mathrm{d}\xi = \frac{8\lambda_m}{Gz}\sum_{m=0}^{\infty} S_{\mathrm{b},m}F'_{\mathrm{b},m}(1)(1-\mathrm{e}^{-\lambda_m}) \quad (35)$$

In Eq. (35) the left-hand side refers to the net outlet energy while the right-hand side is the total amount of heat transfer from hot tubes to the fluid. The results are represented in Figs. 3 and 4 vs. Graetz number with recycle ratio and the channel thickness ratio as parameters, respectively.

Table 4 shows some calculation results of the first two eigenvalues and their associated expansion coefficients as well as the dimensionless outlet temperatures were calculated for $\kappa = 0.5$, $M = 1$, and $Gz = 1$, 10, 100 and 1000. It was found in Table 4 that due to the rapid convergence, only the first negative eigenvalues is necessary to be considered during the calculation of temperature distributions. Fig. 3 shows the relation of another more practical form of dimensionless outlet temperatures θ_F and $\theta_{0,\mathrm{F}}$ vs. Gz with the recycle ratio R as a parameter for $\kappa = 0.5$ while Fig. 4 with the ratio of channel thickness κ as a parameter. It is found in Figs. 3 and 4 that for a fixed recycle ratio, dimensionless outlet temperatures decrease with increasing Gz owing to short residence time of fluid (either large V or small L) but increase with decreasing the ratio of the thickness κ.

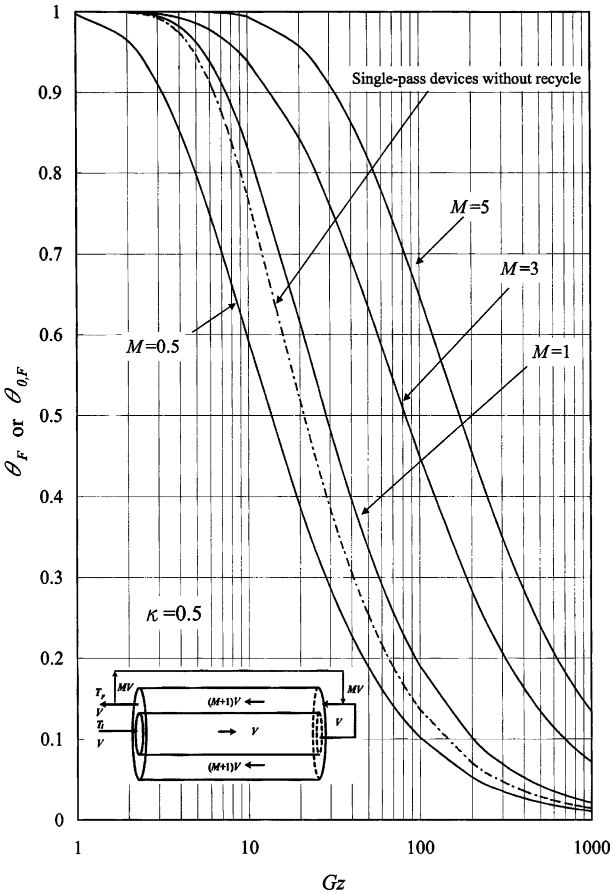

Fig. 3. Dimensionless outlet temperature vs. Gz with reflux ratio as a parameter; $\kappa = 0.5$.

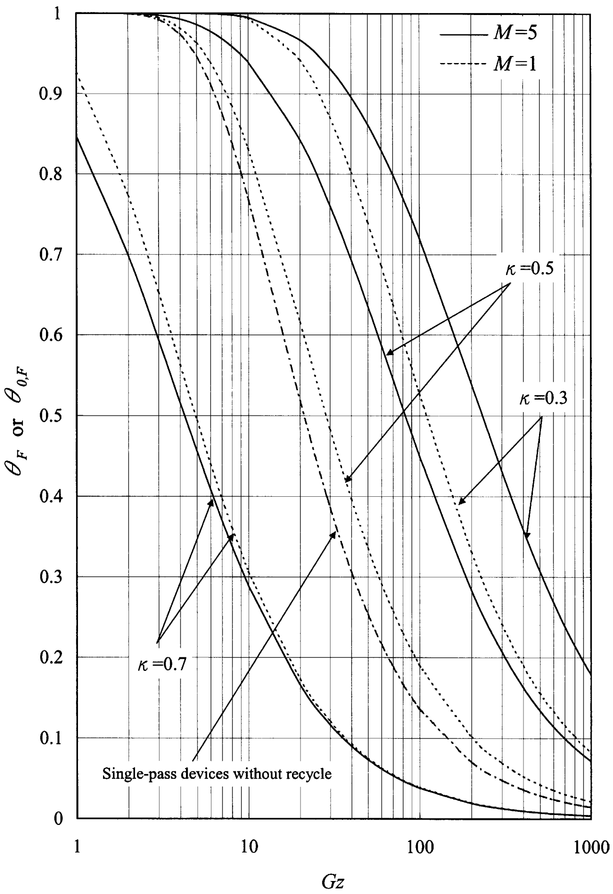

Fig. 4. Dimensionless outlet temperature vs. Gz with κ as a parameter; $M = 1$ and 5.

6.2. Heat transfer efficiencies in double-pass devices

The comparison of average Nusselt numbers, $\overline{Nu}$ and $\overline{Nu}_0$, may be observed from Figs. 5 and 6. Nusselt numbers and hence the heat-transfer efficiency improvement are proportional to $\theta_F(\theta_{0,F})$, as shown in Eqs. (23) and (24), so the higher improvement of performance is really obtained by employing double-pass devices with an impermeable sheet to conduct recycle operations. Figs. 5 and 6 show the average Nusselt numbers with recycle ratio and the channel thickness ratio as parameters, respectively while Table 5 with the channel thickness ratio κ, recycle ratio M and Graetz number Gz as parameters. It is seen from Figs. 5 and 6 and Table 5

Table 4
Eigenvalues and expansion coefficients as well as dimensionless outlet temperatures in double-pass devices with recycle; $\kappa = 0.5$ and $M = 1$, $Gz\lambda_1 = -5.3345$ and $Gz\lambda_2 = -75.882$

Gz	m	λ_m	$S_{a,m}$	$S_{b,m}$	$\psi_F(\lambda_1)$	$\psi_F(\lambda_1, \lambda_2)$
1	1	−5.3345	-5.87×10^{-1}	-2.41×10^{-4}	0.0023	0.0023
	2	−75.822	5.25×10^{-16}	0.0		
10	1	−0.5335	1.47×10^{-1}	-4.17×10^{-2}	0.4070	0.4070
	2	−7.5822	8.42×10^{-16}	0.0		
100	1	−0.0534	1.04	-9.21×10^{-2}	0.8983	0.8983
	2	−0.7582	-3.47×10^{-17}	0.0		
1000	1	−0.0053	1.21	1.01×10^{-1}	0.9891	0.9891
	2	−0.0758	2.36×10^{-15}	0.0		

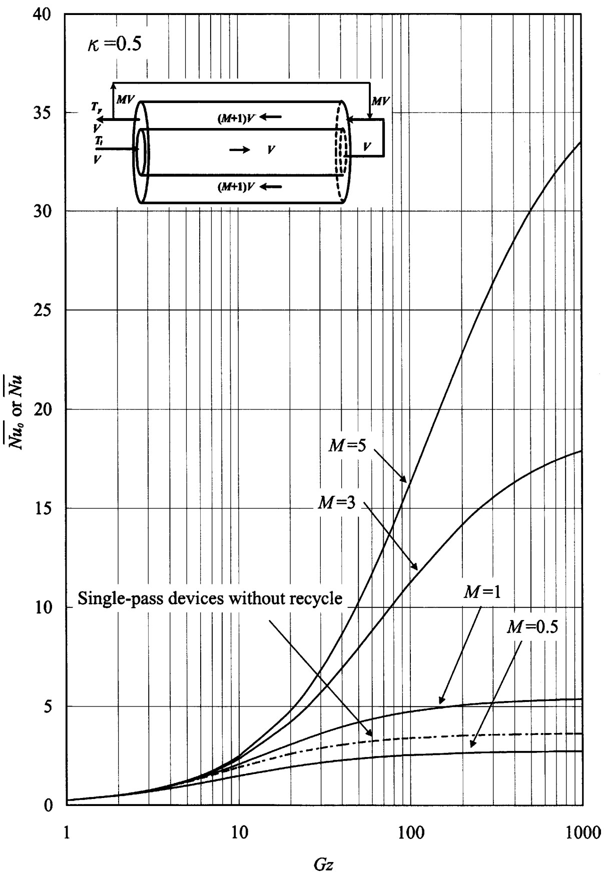

Fig. 5. Average Nusselt number vs. Gz with reflux ratio as a parameter; $\kappa = 0.5$.

that the average Nusselt number increases with Graetz number Gz and recycle ratio M but with decreasing the channel thickness ratio κ.

6.3. Improvement on heat transfer efficiency based single-pass devices

The heat transfer improvement, I_h, in the double-pass operations is shown in Table 5 with Graetz number and the channel thickness ratio κ as parameters. The minus signs in Table 5 indicate that the single-pass device without recycle is preferred rather than double-pass devices operating at such conditions. It is noted that the heat transfer improvement, I_h, increases with Graetz number but with decreasing the channel thickness ratio.

Two case studies were given for the heat-transfer efficiency improvement and the results are shown in Tables 2 and 3. From these two tables we see that I_h increases with Gz and with M. For large L or small V, the larger residence time is the smaller Graetz number would be, and I_h decreases as we proceed down Tables 2 and 3.

7. Conclusion

Orthogonal expansion solutions for parallel-plate exchangers were obtained [17], the theoretical analysis and analytical solution applicable to a broad class of conjugated Graetz problems have not been investigated in detail previously. Further extensions of the Graetz problem to conjugated Graetz problems as well as double-pass countercurrent flow heat exchangers with external refluxes in concentric circular tubes have been developed and solved analytically by the use of the orthogonal expansion technique with the eigenfunction expanding in terms of an extended power series. The mathematical formulation is developed to be particularly useful in the analysis of conjugated boundary value problems with the interfacial boundary condition being not known a priori but being determined as part of the solution to the stated problem. The analysis provides a relatively straightforward strategy to the solution of multistream heat- or mass-transfer devices.

Application of the recycle effect, which is a concept of industrial importance for improving the performance on

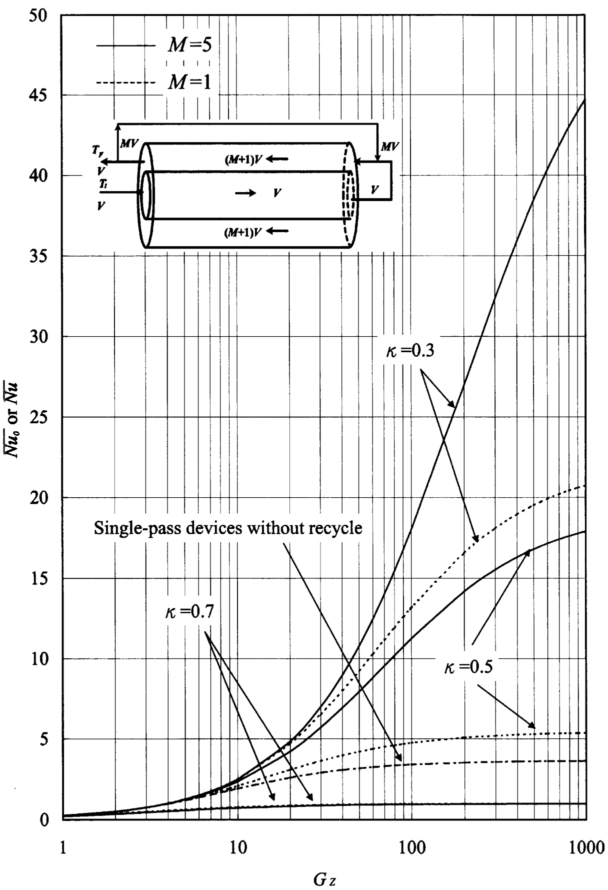

Fig. 6. Average Nusselt number vs. Gz with κ as a parameter; $M = 1$ and 5.

Table 5
The improvement of the transfer efficiency with reflux ratio and channel thickness ratio as parameters

I_h(%)	$M = 0.5$			$M = 1$			$M = 3$			$M = 5$		
	κ			κ			κ			κ		
	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7	0.3	0.5	0.7
$Gz = 1$	0.0	0.2	−7.4	0.0	0.0	−4.7	0.0	0.0	−15.3	0.0	0.0	−13.1
10	29.2	−22.8	−60.3	29.4	8.13	−49.1	29.5	22.2	−62.5	29.5	29.4	−61.9
100	241.9	−25.2	−71.6	287.1	39.6	−71.3	383.3	230.5	−72.2	428.9	276.8	−72.1
1000	351.9	−24.6	−72.7	471.2	47.8	−72.5	850.1	392.9	−73.1	1131	823.3	−73.1

heat transfer, in designing a double-pass heat exchanger is technically and economically feasible. Moreover, further transfer efficiency improvement may be obtained if the smaller channel thickness ratio κ and the larger recycle ratio M are selected. The effects of κ and M on $\overline{Nu}$ are also shown in Figs. 5 and 6 as well as in Table 5. The average Nusselt number $\overline{Nu}$ increases with M and as the value of κ decreases.

It is concluded that the recycle effect can enhance heat transfer for fluid flowing through concentric circular tubes under double-pass operations by inserting an impermeable sheet with negligible thermal resistance. The present study is actually the extension of our previous work [18] except the type of reflux. In order to explain how the present device improves on the previous work, Fig. 7 illustrates the graphical representation for comparisons with the same parameter values used in Fig. 4. With those comparisons, the advantage of present devices is evident for any Graetz numbers.

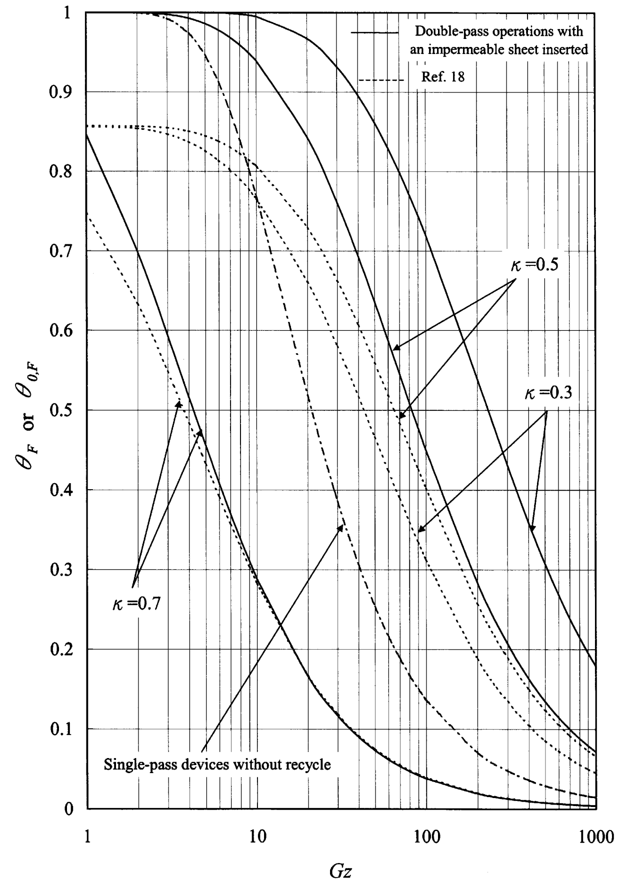

Fig. 7. Dimensionless outlet temperature of both double-pass devices in Ref. [18] and in the present work vs. Gz with κ as a parameter; $M = 5$.

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